Formulas and Applications

It is time to learn how to solve Word Problems (sometimes referred to as Story Problems). Basically it is an 8 step process to solve any word problem, In addition to following those steps you have to make sure that some common knowledge about stuff such as geometry, business, distance formulas, and other things are already known.
The steps are listed below:
Step 1: Read the problem through to determine the type of problem
Step 2: Reread the problem to identify what you are looking for and label the unknowns
Step 3: Let x be the smallest quantity you are looking for and Label the other quantities in terms of x
Step 4: Draw a picture
Step 5: Make a table to represent the data
Step 6: Use the table facts to set up the equation
Step 7: Solve the equation
Step 8: Check to see that your solutions are feasible and do what they are supposed to.

MIXTURE PROBLEMS

PERCENT x	+	PERCENT x	=	PERCENT x
AMOUNT		AMOUNT		AMOUNT

The equation is:
(PERCENT x AMOUNT) + (PERCENT x AMOUNT) = (PERCENT x AMOUNT)

Example1: How many gallons on a 12% salt solution must be combined with a 42% salt solution to obtain 30 gallons of an 18% solution?

	Solution 1	Solution 2	Final Solution
Number of Gallons	x	30-x	30
% of mixture	12%	42%	18%

$0.12x + 0.42(30 - x) = 0.18(30)$
$12x + 42(30 - x) = 18(30)$
$12x + 1260 - 42x = 540$
$-30x = -720$
$x = -720/-30$
$x = 24$
24 gallons of the 12% salt solution must be used.

INVESTMENT PROBLEMS

Investment problems are a type of mixture problems

Example 2: If you have twice as much invested at 8% as at 5% and if your annual interest income from these two investments is $315, how much is invested at each rate?

	First Investment	Second Investment	Total
Annual Interest	5%	8%	
Amount invested	x	2x	
Annual Interest Income	0.05x	0.08(2x)	315

0.05x + 0.08(2x) = 315
5x + 8(2x) = 31500
5x + 16x = 31500
21x = 31500
x = 31500/21
x = 1500
$1500 at 5% and $3000 at 8%

MOTION PROBLEMS

	D	R	T
A			
B			

The table should consist of three columns:
1. GIVEN: You are given the two distances, and the two rates, or the two times.
2. UKNOWN: Read the question to determine this column.
3. FORMULA: D = RT R = D/T T= D/R
The equation comes from the formula column

Example 3: The speed of a stream is 4 m/h. A boat travels 36 miles downstream in the same time it travels 12 miles upstream. Find the speed of the boat in still water.

THREE COLUMNS:

1. GIVEN: You are given the two DISTANCES, 36 miles downstream and 12 miles upstream.
2. UKNOWN: You are asked to find the speed (RATE) of the boat in still water. Let this be x. Keep in mind that all entries in the table apply to the boat in this stream, which is moving at a speed of 4 m/h. Therefore, the rate of the boat going downstream is x + 4 m/h, and the rate of the boat going upstream is x – 4 m/h.
3. FORMULA: The remaining column is the TIME column, and the formula for time is T = D/R; hence, the time going downstream is 36/(x+4) and the time going upstream is 12/(x-4).

	D	R	T
DOWNSTREAM	36	x + 4	36/(x + 4)
UPSTREAM	12	x – 4	12/(x – 4)

Since the equation come from the formula column, we must read the problem again and find a relation between the two times. It says the boat travels downstream and upstream "in the same time," hence

$$\frac{36}{x+4} = \frac{12}{x-4}$$

36(x – 4) = 12(x + 4)
36x – 144 = 12x + 48
36x – 12x = 48 + 144
24x = 192
x = 192/24
x = 8
The speed of the boat in still water is 8 m/h.

WORK PROBLEMS

ALTERNATE METHOD:

	TIME ALONE	RATE	TIME WORKING	PART COMPLETED
A				
B				

The equation comes from the last column. The two parts completed add up to 1 complete job.

Example 4: In a certain post office, Alice can sort a stack of mail in 30 minutes; Bob can sort the same stack in 40 minutes. If they work together, how fast can they sort the stack?

	TIME ALONE	RATE	TIME WORKING	PART COMPLETED
Alice	30	1/30	x	x/30
Bob	40	1/40	x	x/40

x/30 + x/40 = 1
40x + 30x = 120
70x = 120
x = 120/70
x = 17.1 min
Working together, they can sort the stack of mail in about 17 min.

WORK PROBLEMS

Let t_1 = time it takes for the first participant to do the job
t_2 = time it takes for the second participant to do the job etc...
T = time it takes for all the participants to do the job working together

FORMULA: $\frac{1}{t_1}+\frac{1}{t_2}+...=\frac{1}{T}$

Note: If a participant is working against the others, then the fraction 1/t for that participant is negative instead of positive.

Example 5: In a certain post office, Alice can sort a stack of mail in 30 minutes; Bob can sort the same stack in 40 minutes. If they work together, how fast can they sort the stack?

$$\frac{1}{30}+\frac{1}{40}=\frac{1}{x}$$

40x + 30x = 120
70x = 120
x = 120/70
x = 17.1 min

Working together, they can sort the stack of mail in about 17 min.

Using the table method will help us solve any kind of word problems, The examples below are related to different topics

Example 6: The second angle of a triangle is 45° more than the smallest angle. The third angle is three times the smallest. How many degrees are there in each angle?
We are looking for three angles.
$\angle 1$: x
$\angle 2$: x + 45
$\angle 3$: 3x

	$\angle 1$	$\angle 2$	$\angle 3$	Total
Angle measure	x	x+45	3x	180

You would not be able to solve this problem unless you knew that the sum of the interior angles of a triangle is 180°.
$\angle 1 + \angle 2 + \angle 3 = 180°$
x + x + 45 + 3x = 180°
5x + 45 = 180°
5x = 135°
x = 27°
That means the first angle is 27°, the second angle is 27 + 45 or 72°, and the third angle is 3 times 27 or 81°.

Example 7: Two trains start from the same station at the same time and travel in opposite directions. One train travels at an average rate of 40 mph, the other at 65 mph. In how many hours will they be 315 miles apart?

First we'll make the d=rt chart.
Let x be the time needed for the two trains to be 315 miles.

d = r x **t**
Train 1: 40 x
Train 2: 65 x

	Train 1	Train 2	Total
Time	x	x	
Rate	40	65	
Distance	40x	65x	315

The reason we have an x in the time column is because they left at the same time and will be 315 at the same time. In other words, their times are equal.

40x + 65x = 315
105x = 315
x = 3
It will take three hours.

Example 8: Mr. Williams starts out in his auto traveling 30 miles per hour. Four hours later Mr. Speedster starts out from the same point at 60 miles per hour to overtake Mr. Williams. In how many hours will he be overtaken?
Each will have traveled the same distance when they meet. x =Speedster's time.
R x T = D

	Speed	Time Needed	Distance
William	30	x+4	30(x+4)
Speedster	60	x	60x

When they will overtake, they will be covering the same distance

$\Rightarrow 30(x+4) = 60x$

$\Rightarrow 30x + 120 = 60x$

$\Rightarrow 30x = 120$

$x = 4$