Factoring Polynomial Functions

Factoring is writing an expression as a product of factors. For example, we can write 10 as (5)(2), where 5 and 2 are called factors of 10. We can also do this with polynomial expressions, there are different ways to do the factorization process, in this section we will remember the basic factorization methods

1) Factor out the GCF of a polynomial.

The GCF for a polynomial is the largest monomial that divides (is a factor of) each term of the polynomial.

Step 1: Identify the GCF of the polynomial.
Step 2: Divide the GCF out of every term of the polynomial.

Example 1: Factorize: $P(z) = 90z^6 - 5z^4 + 25z^2$

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$\text{The GCF is } 5z^2$

$\Rightarrow P(z) = 5z^2\left(18z^4 - z^2 + 5\right)$

2) Factor a polynomial with four terms by grouping.

In some cases there is not a GCF for ALL the terms in a polynomial. If you have four terms with no GCF, then try factoring by grouping.

Step 1: Group the first two terms together and then the last two terms together.
Step 2: Factor out a GCF from each separate binomial.
Step 3: Factor out the common binomial

Example 2: Factorize: $P(x) = 7x^3 - 14x^2 - x + 2$.

$P(x) = 7x^3 - 14x^2 - x + 2$

$\Rightarrow P(x) = \left(7x^3 - 14x^2\right) - (x-2)$

$\Rightarrow P(x) = 7x^2(x-2) - (x-2)$

$\Rightarrow P(x) = 7x^2(x-2) - 1(x-2)$

$\Rightarrow P(x) = \left(7x^2 - 1\right)(x-2)$

3) Factor a trinomial of the form $x^2 + bx + c$.

Basically, we are reversing the FOIL method to get our factored form. We are looking for two binomials that when you multiply them you get the given trinomial.

Step 1: Set up a product of two ( ) where each will hold two terms.
Step 2: Find the factors that go in the first positions.
Step 3: Find the factors that go in the last positions.
The factors that would go in the last position would have to be two expressions such that their **product equals *c*** (the constant) and at the same time their **sum equals *b*** (number in front of *x* term).
As you are finding these factors, you have to consider the sign of the expressions:
If *c* is positive, your factors are going to both have the same sign depending on *b*'s sign.
If *c* is negative, your factors are going to have opposite signs depending on *b*'s sign.

Example 3: Factorize:

1) $P(x) = x^2 + 6x + 8$

$$P(x) = x^2 + 6x + 8$$
$$\Rightarrow P(x) = x^2 + (4+2)x + (4\times 2)$$
$$\Rightarrow P(x) = (x+2)(x+4)$$

2) $P(x) = x^2 + 5x - 24$

$$P(x) = x^2 + 5x - 24$$
$$\Rightarrow P(x) = x^2 + (8-3)x + (8\times(-3))$$
$$\Rightarrow P(x) = (x+8)(x-3)$$

4) Factor a trinomial of the form $ax^2 + bx + c$.

Again, this is the reverse of the FOIL method.
The difference between this trinomial and the one discussed above, is the coefficient of x^2, here it is a number other than 1. **This means, that not only do you need to find factors of c, but also *a*.**
Step 1: Set up a product of two ( ) where each will hold two terms.
Step 2: Use trial and error to find the factors needed.
The factors of *a* will go in the first terms of the binomials and the factors of *c* will go in the last terms of the binomials.
The trick is to get the right combination of these factors. You can check this by applying the FOIL method. If your product comes out to be the trinomial you started with, you have the right combination of factors. If the product does not come out to be the given trinomial, then you need to try again.

Example 4: Factorize: $P(x) = 3x^2 + 11x - 4$

$$P(x) = 3x^2 + 11x - 4$$
$$\Rightarrow P(x) = (3x-1)(x+4)$$

Factor a perfect square trinomial. $(a+b)^2 = a^2 + 2ab + b^2$ or $(a-b)^2 = a^2 - 2ab + b^2$

It has to be exactly in this form to use this rule. When you have a base being squared plus or minus twice the product of the two bases plus another base squared, it factors as the sum (or difference) of the bases being squared.

Example 5: Factorize:

1) $P(x) = x^2 + 18x + 81$

$P(x) = x^2 + 18x + 81$

$\Rightarrow P(x) = x^2 + 2(9)x + (9)^2$

$\Rightarrow P(x) = (x+9)^2$

2) $P(x) = x^2 - 16x + 64$

$P(x) = x^2 - 16x + 64$

$\Rightarrow P(x) = x^2 + 2(8)x + (8)^2$

$\Rightarrow P(x) = (x-8)^2$

6) **Factor a difference of squares.** $a^2 - b^2 = (a-b)(a+b)$

Just like the perfect square trinomial, the difference of two squares has to be exactly in this form to use this rule. When you have the difference of two bases being squared, it factors as the product of the sum and difference of the bases that are being squared.
Note that the sum of two squares DOES NOT factor.

Example 6: Factorize: $P(x) = x^2 - 121$

$P(x) = x^2 - 121$

$\Rightarrow P(x) = x^2 - (11)^2$

$\Rightarrow P(x) = (x-11)(x+11)$

7) **Factor a sum or difference of cubes.**

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$: The sum of two cubes has to be exactly in this form to use this rule. When you have the sum of two cubes, you have a product of a binomial and a trinomial. The binomial is the sum of the bases that are being cubed. The trinomial is the first base squared, the second term is the opposite of the product of the two bases found, and the third term is the second base squared.

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$: This is factored in a similar fashion to the sum of two cubes. Note the only difference is that the sign in the binomial is "a" - which matches the original sign, and the sign in front of "ax" is positive, which is the opposite sign. The difference of two cubes has to be exactly in this form to use this rule.

Example 7: Factorize: $P(x)=x^3+27$.

$P(x)=x^3+27$

$\Rightarrow P(x)=x^3+(3)^3$

$\Rightarrow P(x)=(x+3)(x^2-3x+9)$

Remark: Not every polynomial is factorable. Just like not every number has a factor other than 1 or itself. A prime number is a number that has exactly two factors, 1 and itself. 2, 3, and 5 are examples of prime numbers.
The same thing can occur with polynomials. If a polynomial is not factorable we say that it is a **prime polynomial**.

Warning! Do not take the decision of writing it is **prime** unless you have tried all the possibilities to factor it.