Equations Reducible to Quadratics

Rule 1: Solving polynomial equations by factoring

Example 1: Solve:

$3x^4 = 48x^2$	Write the equation in standard form (zero on the right side).
$3x^4 - 48x^2 = 0$	Take out the common factor and collect the remaining factor.
$3x^2(x^2 - 16) = 0$	Factor completely.
$3x^2(x-4)(x+4) = 0$	Set each factor equal to zero.
$3x^2 = 0$ or $x - 4 = 0$ or $x + 4 = 0$	Solve for x.
$x = 0$ & $x = 4$ & $x = -4$	

Rule 2: Solving equations involving rational exponents

The first step is to isolate the rational expression. Second, determine the rational exponent and raise both sides of the equation to the reciprocal exponent. Simplify and check your answers.

Example 2: Solve:

$4x^{3/2} - 8 = 0$	
$4x^{3/2} = 8$	Add 8 to both sides of the equation.
$x^{3/2} = 2$	Divide both sides by 4.
$(x^{3/2})^{2/3} = (2)^{2/3}$	Raise both sides to the 2/3 power(reciprocal power).
$x = 2^{2/3}$ OR $x = \sqrt[3]{4}$	

Check:

$4x^{3/2} - 8$	
$4(2^{2/3})^{3/2} - 8$	Replace x by $2^{2/3}$.
$4(2) - 8$	Simplify.
$8 - 8$	
0	The statement is true; therefore, the solution checks.

Rule 3: Solving Radical Equations:

The basic approach to solving radical equations is to get rid of the radical equation. Get rid of the square root by squaring each side of the equation. Get rid of the cube root by cubing both sides of the equation, etc.

In order to solve a radical equation in one variable algebraically, we need to know the **principle of powers** rule.

Principle of Powers: If $a=b$, then $a^n=b^n$.

When changing radical equations and those equations involving rational exponents we often get extraneous solutions (solutions that do not fit the original equation). Therefore, a check of each tentative solution is required.

To solve an equation algebraically using the principle of powers:

Step 1: Isolate a radical term on one side of the equation.
Step 2: Raise both expressions in the equation to the power of the index of the isolated term.
Step 3: Solve the resulting equation.
Step 4: If a radical term remains, perform steps 1 – 3 again
Step 5: Check your solution(s) by substitution.

Example 3: Solve the radical equations and check all tentative solutions:

$$\sqrt{2x+7}-x=2$$
$$\sqrt{2x+7}=2+x$$
$$\left(\sqrt{2x+7}\right)^2=(2+x)^2$$
$$2x+7=x^2+4x+4$$
$$x^2+2x-3=0$$
$$(x+3)(x-1)=0$$
$$x+3=0 \qquad x-1=0$$
$$x=-3 \qquad x=1$$

Check:

$$\sqrt{2(-3)+7}-(-3)$$
$$=\sqrt{1}+3=1+3=4\neq 2$$

Therefore, -3 is not a solution

$$\sqrt{2(1)+7}-(1)$$
$$=\sqrt{9}-1=3-1=2$$

Therefore, 1 is a solution

Example 4: Solve $\sqrt[5]{3x+4}=2$

$$\left(\sqrt[5]{3x+4}\right)^5=(2)^5$$
$$3x+4=32$$
$$3x=28$$
$$x=\frac{28}{3}$$

Rule 4: **Solving Equations involving fractions:**
To solve an equation involving fractions, multiply each term in the equation by the least common denominator. This procedure will clear the equation of fractions.

Example 5: Solve the following equation that contain fractions and check the tentative solutions:

$\frac{2}{x^2+1}+\frac{1}{x}=\frac{2}{x}$ Multiply each term of the equation by the common denominator $x\left(x^2+1\right)$

$$\frac{2}{x^2+1}+\frac{1}{x}=\frac{2}{x}$$
$$\Rightarrow x\left(x^2+1\right)\left(\frac{2}{x^2+1}\right)+x\left(x^2+1\right)\left(\frac{1}{x}\right)=x\left(x^2+1\right)\left(\frac{2}{x}\right)$$
$$\Rightarrow 2x+1\left(x^2+1\right)=2\left(x^2+1\right)$$
$$\Rightarrow 2x+x^2+1=2x^2+2$$
$$\Rightarrow x^2-2x+1=0$$
$$\Rightarrow (x-1)(x-1)=0$$
$$\Rightarrow x-1=0$$
$$x=1$$

Check:

$$\frac{2}{1^2+1}+\frac{1}{1}\overset{?}{=}\frac{2}{1}$$
$$\Rightarrow \frac{2}{2}+1\overset{?}{=}2$$

Therefore, x=1 is a solution

$$1+1\overset{?}{=}2$$
$$2=2$$

Example 6: Solve:

$$\frac{1}{t}+\frac{1}{2t}+\frac{1}{3t}=5$$

$$6t\left(\frac{1}{t}+\frac{1}{2t}+\frac{1}{3t}\right)=(5)6t$$
$$6t\left(\frac{1}{t}\right)+6t\left(\frac{1}{2t}\right)+6t\left(\frac{1}{3t}\right)=30t$$
$$6(1)+3(1)+2(1)=30t$$
$$11=30t$$
$$30t=11$$
$$t=\frac{11}{30}$$

Rule 5: **Solving equations involving absolute values:**

To solve an equation involving an absolute value, consider the fact that the expression inside the absolute value can be positive or negative. This consideration results in two separate equations, each of which must be solved:

Example 7: Solve the equation:

$|x-2|=3$

$x-2=3$ OR $-(x-2)=3$

$x=5$ $\quad -x+2=3$

$-x=1$

$x=-1$

Check:

$|5-2|=3$ OR $|-1-2|=3$

$|3|=3$ $\quad |-3|=3$

$3=3$ $\quad 3=3$

Example 8: Solve the equation: $|x^2-2x|=3$

$|x^2-2x|=3$

$x^2-2x=3$ OR $-(x^2-2x)=3$

$x^2-2x-3=0$ $\quad -x^2+2x-3=0$

$(x-3)(x+1)=0$ $\quad \Delta=4-12=-8<0$

$x=3 \quad and \quad x=-1$