Domain and Range

A function is generally defined as a rule that partners to each number *x* in a set a unique number *y* in another set. The set of *x* values to which the rule applies is the function's **domain**, and the set of *y* values to which it applies is its **range**.

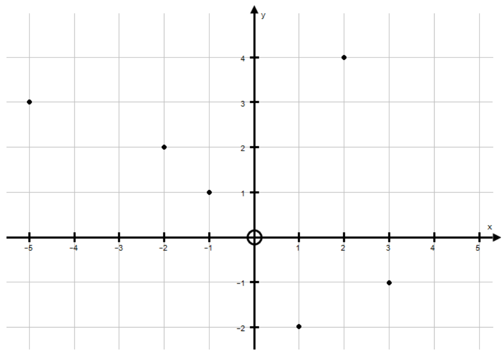
The domain and range of a function are the essence or foundation of algebraic equations and calculus formulas. Everyday uses include graphs, charts and maps.
Fundamental Mathematical Concepts and Terms

A function is a set of ordered pairs (*x*,*y*) such that for each first element *x*, there always corresponds one and only one element *y*. The domain is the set of the first elements and the range is the term given to name the set of the second elements. The domain is referred to as the independent variable and the range as the dependent variable.

The domain is the first group or set of values being fed or input into a function and these values will serve as the x-axis of a graph or chart.

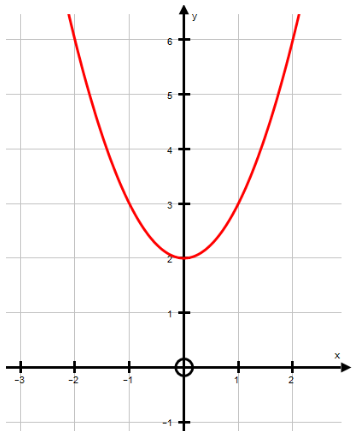
The range is the second group or set of values being fed or input into a function with these values serving as the y-axis of a graph or chart.
The domain and range can be clearly identified graphically

Example 1: State the domain and the range of
$\{(-1,1);(-2,2);(-5,3);(2,4);(3,-1);(1,-2)\}$
The points are plotted as shown.
The domain is the set of x values
$\Rightarrow Domain:\{-5,-1,-2,1,2,3\}$
The range is the set of y values
$\Rightarrow Range:\{-1,-2,1,2,3,4\}$

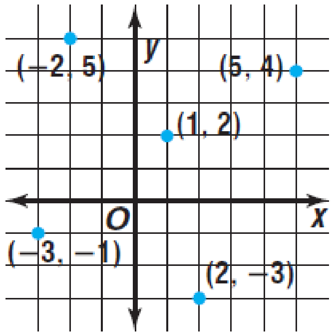


Example 2: Find graphically the domain and the range of the function

The domain is the set of x values, as shown x can be positive, negative or zero
$\Rightarrow Domain\ is\ \mathbb{R}$
The range is the set of y values. The graph has a minimum y value that is equal to 2
$\Rightarrow Range\ is\ \ y \geq 2$
$\Rightarrow Range:\ y \in [2,+\infty)$



Example 3: Express the relation shown on the graph as a set of ordered pairs and in a table. Then determine the domain and range.



The set of ordered pairs for the relation is {(-3, -1), (-2, 5), (1, 2), (2, -3), (5, 4)}.
As a table:

x	y
−3	−1
−2	5
1	2
2	−3
5	4

The domain is {-3, -2, 1, 2, 5}, and the range is {-1, 5, 2, -3, 4}.

Finding Domains: There are two cases which require a discussion for x to find the domain of the function:

1. If the function has division, find out where the denominator will equal zero.

For example: $f(x)=\dfrac{2x+7}{x-3}$ has no value at x = 3 because the denominator equals zero. So the domain is all real numbers except 3.

2. If the function has a square root, find out where the square root part will have a negative value under the sign and eliminate all the x values that will make the expression negative.

For example: $f(x)=\sqrt{3x-6}$. The expression under the root is $3x-6$.
$3x-6\geq 0 \Rightarrow x\geq 2$. So, the domain is $x\in[2,+\infty)$

Finding Ranges: To find the range of a function, plug in the domain. Find the minimum and maximum. If the function can become indefinitely big, it will be to positive infinity. Similarly, if it can become indefinitely small, it will be to negative infinity.

We can easily find the range of a function if any of the two following cases are applicable:

1. If it has only absolute values or squares and square roots, it can never be negative
2. If it is a quotient and the numerator is constant, it can never be equal to 0

Example 4: Find the domain of $y = 2x - 5$ if the range is {-3, -1, 1, 3}.
Make a table. Substitute each value of *y* into the equation. Then solve each equation to determine the corresponding values of *x*.

y	$y = 2x - 5$	x	(x, y)
-3	$-3 = 2x - 5$	1	$(1, -3)$
-1	$-1 = 2x - 5$	2	$(2, -1)$
1	$1 = 2x - 5$	3	$(3, 1)$
3	$3 = 2x - 5$	4	$(4, 3)$

The domain is {1, 2, 3, 4}.

Example 5: Find the domain and the range of each function

1) $f(x) = 2x + 4$

Domain:
There is no denominator and there is no square root sign
$\Rightarrow Domain\ is\ \mathbb{R}$
Range:
For every value of x there corresponds a value of y, x can be any real number and so as y
$\Rightarrow Range\ is\ \mathbb{R}$

2) $f(x) = \sqrt{x}$

Domain:
The expression under the root sign must be positive
$\Rightarrow x \geq 0$
$\Rightarrow$The domain is all positive numbers
Range:
The square root value is always positive
$\Rightarrow$ The range is all positive numbers i.e. $x \in [0, +\infty)$

3) $f(x) = -\sqrt{-x-4}$

Domain:

The expression under the root sign must be positive

$\Rightarrow -x-4 \geq 0$

$\Rightarrow -x \geq 4$

$\Rightarrow x \leq -4$

$\Rightarrow$The domain is $x \in (-\infty, -4]$

Range:

The square root value is always positive

$\Rightarrow$ The range is $y \in (-\infty, 0]$

4) $f(x) = \dfrac{7}{x+3}$

Domain:

The denominator cannot be equal to zero

$\Rightarrow x+3 \neq 0$

$\Rightarrow x \neq -3$

$\Rightarrow$The domain is all real numbers except -3 i.e. $x \in \mathbb{R} - \{-3\}$

Range:

It is a quotient and the numerator is constant, it can never be equal to 0

$\Rightarrow$The range is $y \in \mathbb{R} - \{0\}$