Division of Polynomial Functions

Long Division of Polynomials

Long division of polynomials is a lot like long division of real numbers. If the polynomials involved were written in fraction form, the numerator would be the dividend, and the denominator would be the divisor.

Rule 1: To find the quotient of two polynomials using long division we follow the steps listed below:

Step 1: Write dividend and divisor polynomials in standard polynomial form. Use zero coefficients for powers of the variable which are missing in the dividend and divisor.

Step 2: Divide first term of the divisor into the first term of the dividend. Put this quotient above term in the dividend.

Step 3: Multiply quotient by all terms of the divisor and put products under the appropriate terms of the dividend.

Step 4: Subtract (change signs on bottom polynomial and add) and bring down remaining terms.

Step 5: Continue to divide first term by first term until the power of the divisor is larger than the power in the dividend.

Example 1: Find the quotient and the remainder of:

$$\frac{2x^2+x-10}{x-2}$$

- Write as a division problem, making sure both polynomials are in descending powers of exponents.

$$x-2\overline{)2x^2+x-10}$$

- Divide the first term of $2x^2+x-10$ by the first term of $x-2$

$$\begin{array}{r} 2x \\ x-2\overline{)2x^2+x-10} \end{array}$$

- Now multiply $x-2$ by $2x$ and place the result underneath, just like normal division.

$$\begin{array}{r} 2x \\ x-2\overline{)2x^2+x-10} \\ \underline{2x^2-4x} \end{array}$$

- Now subtract the polynomials.

$$\begin{array}{r} 2x \\ x-2\overline{)2x^2+x-10} \\ \underline{2x^2-4x} \\ 5x-10 \end{array}$$

- Now divide the first term of $5x-10$ by the first term of $x-2$.

$$\begin{array}{r|l} & 2x+5 \\ \hline x-2 & 2x^2+x-10 \\ & \underline{2x^2-4x} \\ & \quad 5x-10 \end{array}$$

- Now multiply 5 by $x-2$ and place the result underneath.

$$\begin{array}{r|l} & 2x+5 \\ \hline x-2 & 2x^2+x-10 \\ & \underline{2x^2-4x} \\ & \quad 5x-10 \\ & \quad \underline{5x-10} \\ & \qquad 0 \end{array}$$

At this point you are finished. The remainder is 0. You should check your answer by multiplying $2x+5$ times $x-2$. Your answer should be $2x^2+x-10$.

Example 2: $f(x)=-x^3+3x+8$ is divided by $g(x)=x-2$

$$\begin{array}{r|l} & -x^2-2x-1 \\ \hline x-2 & -x^3+3x+8 \\ & \underline{-x^3+2x^2} \\ & 0-2x^2+3x+8 \\ & \underline{-2x^2+4x} \\ & \qquad -x+8 \\ & \qquad \underline{-x+2} \\ & \qquad\quad 6 \end{array}$$

$(-x^3+3x+8)\div(x-2)=-x^2-2x-1+\dfrac{6}{x-2}$ **OR** $-x^3+3x+8=(-x^2-2x-1)(x-2)+6$

Rule 2: **The Division Algorithm**: If $P(x)$ and $D(x)$ are polynomials such that $D(x)\neq 0$, and the degree of $D(x)$ is less than or equal to the degree of $P(x)$, there exist unique polynomials $Q(x)$ and $R(x)$ such that:

$$\underset{\text{Dividend}}{P(x)} = \underset{\text{Divisor}}{D(x)} \times \underset{\text{Quotient}}{Q(x)} + \underset{\text{Remainder}}{R(x)}$$

Where $R(x)=0$ or the degree of $R(x)$ is less than the degree of $D(x)$. If the remainder $R(x)$ is zero, $D(x)$ divides evenly into $P(x)$.

The division algorithm can also be written as:

$$\frac{P(x)}{D(x)}=Q(x)+\frac{R(x)}{D(x)}$$

Remark 1: $Q(x)$ and $R(x)$ are unique, or in other words, there is only one $Q(x)$ and $R(x)$ that will work for a given $P(x)$ and $D(x)$.

Synthetic division

Synthetic division is a "shortcut" method of performing polynomial long division. We can use synthetic division when we want to divide a polynomial f(x) by a linear divisor of the form $x-a$, where the coefficient of x is 1. Since *a* can be positive, negative or zero, the divisor can look like:

- x
- $x-a$
- $x+a$

Remark 2: Those are the only cases we can use synthetic division. If we are not dividing by a linear divisor of the form $x-a$, then we need to use polynomial long division.

Synthetic division requires only three lines:
1) The top line for the dividend and divisor
2) The second line for the intermediate values
3) The third line for the quotient and remainder

Rule 3: To find the quotient of two polynomials using synthetic division we follow the steps listed below. Let the dividend have degree n:

Step 1: In line one, write the coefficients of the polynomial as the dividend, and let c be the divisor. (The coefficients should be for x when the polynomial function is arranged in descending order.)

Step 2: In line three rewrite the leading coefficient of the dividend directly below its position in the dividend; multiply it by the divisor, and write the product in line two directly below the coefficient of x^{n-1}.

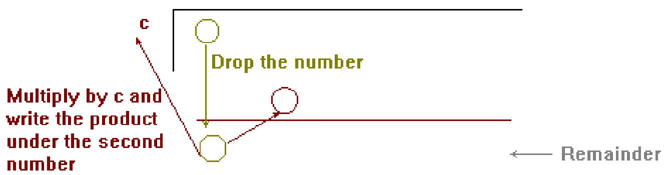
Step 3: Add this product to the number directly above it in the dividend (this number is the coefficient of x^{n-1}) to get a new number.

Repeat steps two and three until the entire polynomial has been divided.

The quotient will be one degree less than the dividend.

The coefficients of the quotient are the first (n – 1) numbers in line three.

The remainder is the last number in line three.



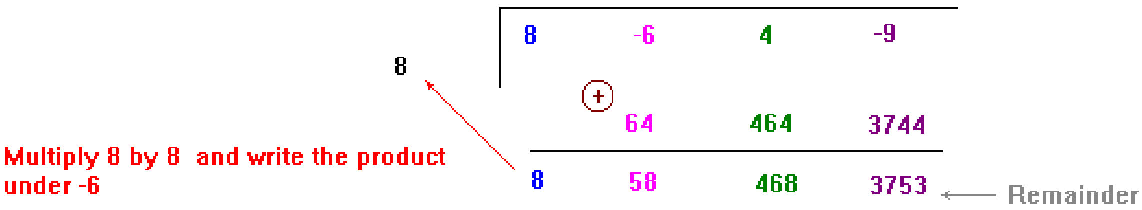
Example 3: Find the quotient of $f(x) = 8x^3 - 6x^2 + 4x - 9$ by $g(x) = x - 8$

Using Long Division:

$$
\begin{array}{l}
\qquad\quad 8x^2 + 58x + 468 \\
x-8\overline{)8x^3 - 6x^2 + 4x - 9} \\
\qquad \underline{8x^3 - 64x^2} \\
\qquad 0 + 58x^2 + 4x - 9 \\
\qquad \underline{....+58x^2 - 464x} \\
\qquad ....0 + 468x - 9 \\
\qquad \underline{............468x - 3744} \\
\qquad ....0 + 3753
\end{array}
$$

$$\left(8x^3 - 6x^2 + 4x - 9\right) \div (x - 8) = 8x^2 + 58x + 468 + \frac{3753}{x-8}$$

Using Synthetic Division:



$$\left(8x^3 - 6x^2 + 4x - 9\right) \div (x - 8) = 8x^2 + 58x + 468 + \frac{3753}{x-8}$$

Remark 3: If we are using synthetic division and the divisor has missing degrees of x then we write zeros to fill the missing coefficients, i.e. $\left(x^4 - x + 1\right) \div (x - 2)$, we represent it as follows:

$$2 \,\Big|\; 1 \quad 0 \quad 0 \quad -1 \quad 1$$

then we continue...