Cramer's Rule

Determinants can be used to solve n equations in n variables. The method used is called Cramer's rule and is illustrated below. If $n \geq 4$ then a special calculator or your computer is a better tool.

Rule 1: Cramer's Rule.

The solution (*x*, *y*) of the system $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$ can be found using determinants:

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad \& \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Example 1: Solve the system using Cramer's Rule: $\begin{cases} x - 3y = 6 \\ 2x + 3y = 3 \end{cases}$

First we determine the values we will need for Cramer's Rule:

$a_1 = 1$ $b_1 = -3$ $c_1 = 6$

$a_2 = 2$ $b_2 = 3$ $c_2 = 3$

$$x = \frac{\begin{vmatrix} 6 & -3 \\ 3 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix}} = \frac{18+9}{3+6} = 3$$

$$y = \frac{\begin{vmatrix} 1 & 6 \\ 2 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix}} = \frac{3-12}{3+6} = \frac{-9}{9} = -1$$

So the solution is (3, -1).

Check: [1] 3 + 3 = 6

[2] 6 - 3 = 3