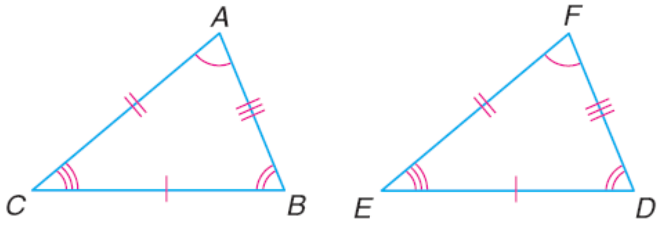
Congruent Triangles

You know that congruent segments have the same length and congruent angles have the same degree measure. In the following activity, you will learn about congruent triangles are related.

Activity 1:
Step 1: On a piece of grid paper, draw two triangles like the ones below.
Step 2: Cut out the triangles. Put one triangle over the other so that the parts with the same measures match up.
1. Identify all of the pairs of angles and sides that match or correspond.
2. $\triangle ABC$ is congruent to $\triangle FDE$. What is true about their corresponding sides and angles?

If a triangle can be translated, rotated, or reflected onto another triangle so that all of the vertices correspond, the triangles are **congruent triangles**. The parts of congruent triangles that "match "are called **corresponding parts**.

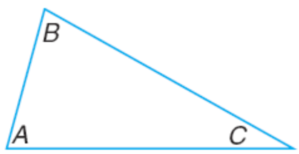


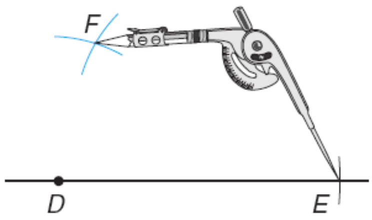
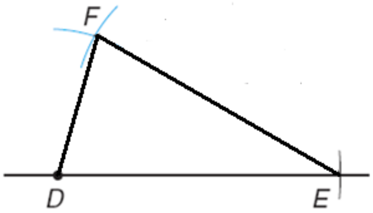
Corresponding Angles — Corresponding Sides

$$\triangle ABC \cong \triangle FDE \Leftrightarrow \left.\begin{matrix} \angle CAB \cong \angle EFD \\ \angle ACB \cong \angle FED \\ \angle ABC \cong \angle FDE \end{matrix}\right\} \;\&\; \left\{\begin{matrix} \overline{AC} \cong \overline{FE} \\ \overline{AB} \cong \overline{FD} \\ \overline{CB} \cong \overline{ED} \end{matrix}\right.$$

Definition 1: **Definition of Congruent Triangles:**

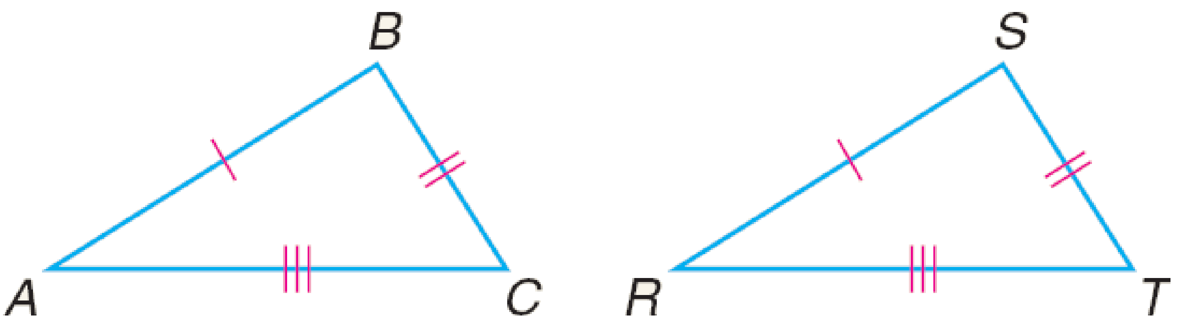
If the corresponding parts of two triangles are congruent, then the two triangles are congruent.
If two triangles are congruent, then the corresponding parts of the two triangles are congruent.
Activity 2: Construct $\triangle DFE$; given $\triangle ABC$ and $\overline{AB} \cong \overline{DF}, \overline{BC} \cong \overline{FE}, \overline{AC} \cong \overline{DE}$

Step 1: Draw an acute scalene triangle on a piece of paper. Label its vertices *A*, *B*, and *C* on the interior of each angle.	
Step 2: Construct a segment congruent to $\overline{AC}$. Label the endpoints of the segment *D* and *E*.	
Step 3: Adjust the compass setting to the length of $\overline{AB}$. Place the compass at point *D* and draw a large arc above $\overline{DE}$.	

Step 4: Adjust the compass setting to the length of $\overline{CB}$. Place the compass at point *E* and draw an arc above DE to intersect the one drawn from point *D*. Label the intersection *F*.	
Step 5: Draw $\overline{DF}$ and $\overline{EF}$.	

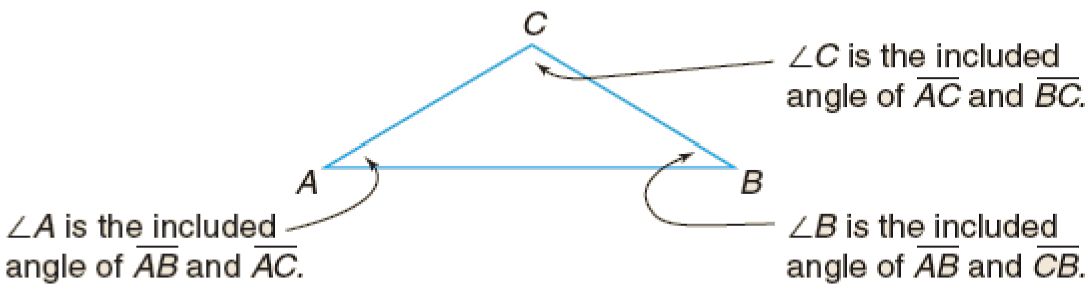
In the previous activity, you constructed a congruent triangle by using only the measures of its sides. This activity suggests the following theorem.

Postulate 1: Side – Side – Side Postulate. "SSS Postulate": If three sides of one triangle are congruent to three corresponding sides of another triangle, then the triangles are congruent.

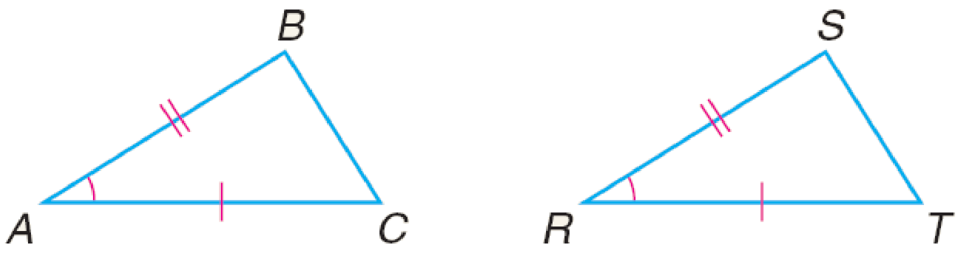


If $\overline{AC} \cong \overline{RT}, \overline{AB} \cong \overline{RS}, \overline{CB} \cong \overline{TS}$ then $\triangle ABC \cong \triangle RST$

Definition 2: In a triangle, the angle formed by two given sides is called the **included angle** of the sides.

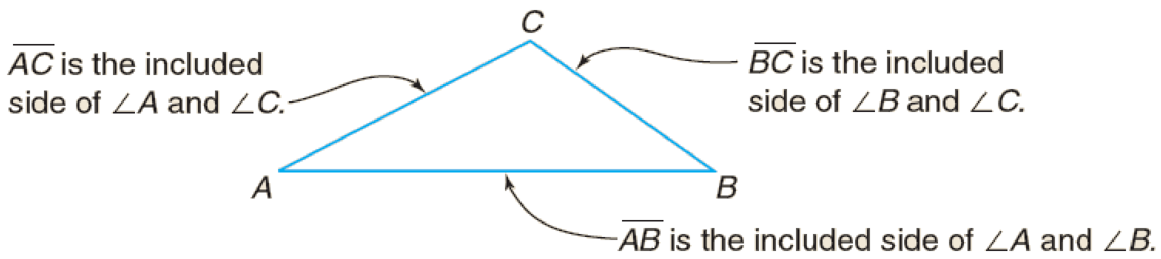


Postulate 2: Side – Angle – Side Postulate. "SAS Postulate": If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the triangles are congruent.

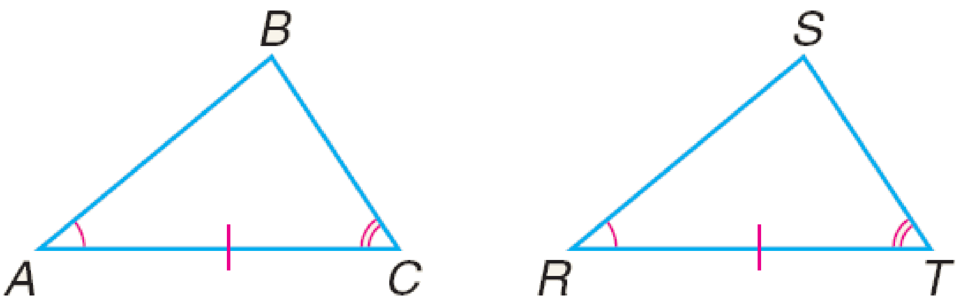


If $\overline{AC} \cong \overline{RT}, \angle BAC \cong \angle SRT, \overline{AB} \cong \overline{RS}$ then $\triangle ABC \cong \triangle RST$

Definition 3: The side of a triangle that falls between two given angles is called the **included side** of the angles. It is the one side common to both angles.

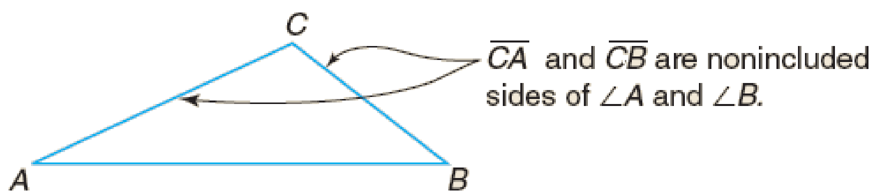


Postulate 3: Angle – Side – Angle Postulate. "ASA Postulate": If two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle, then the triangles are congruent.

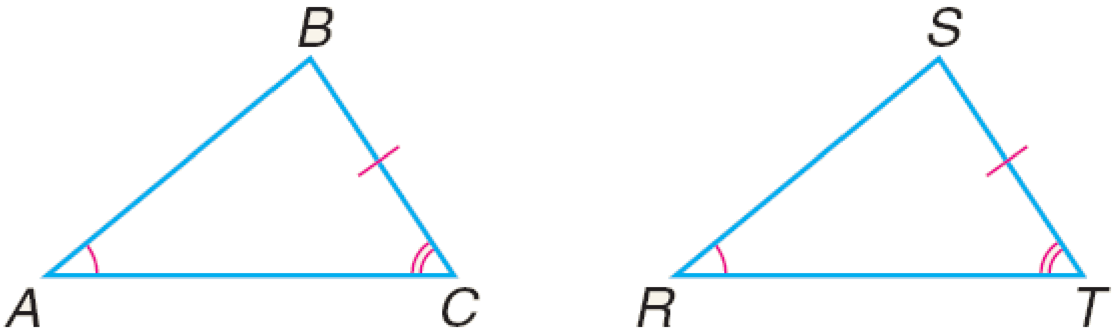


If $\angle BCA \cong \angle STR, \overline{AC} \cong \overline{RT}, \angle BAC \cong \angle SRT$ then $\triangle ABC \cong \triangle RST$

The Angle-Angle-Side Theorem is derived from the ASA Postulate. In AAS, the S is *not* between the two given angles. Therefore, the S indicates a side that is not included between the two angles.



Theorem 1: Angle – Angle – Side Theorem. "AAS Theorem": If two angles and a non-included side of one triangle are congruent to the corresponding two angles and non-included side of another triangle, then the triangles are congruent.



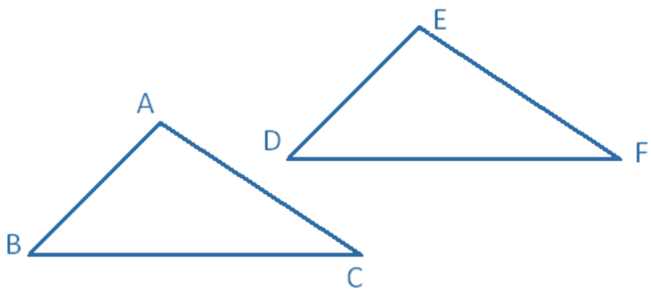
If $\angle BCA \cong \angle STR, \angle BAC \cong \angle SRT, \overline{CB} \cong \overline{TS}$ then $\triangle ABC \cong \triangle RST$

Example 1:

Given: $\angle ABC \cong \angle EDF$
$EF = AC$
$AB = DE$

Prove: $\triangle ABC \cong \triangle EDF$

Proof:



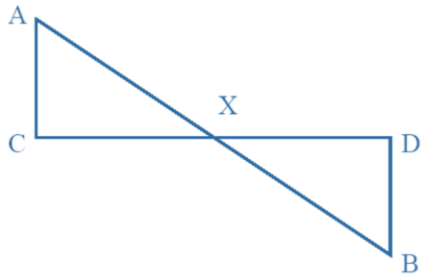
Statements	Reasons
In $\triangle ABC$ and $\triangle EDF$, we have:	
1) $AB = DE$	1) Given
2) $EF = AC$	2) Given
3) $\angle ABC \cong \angle EDF$	3) Given
4) $\triangle ABC \cong \triangle EDF$	4) SAS Postulate

Example 2:

Given: $\angle ACX \cong \angle BDX$
$BD = AC$

Prove: $\triangle ACX \cong \triangle BDX$

Proof:



Statements	Reasons
In $\triangle ACX$ and $\triangle BDX$, we have:	
1) $BD = AC$	1) Given
2) $\angle ACX \cong \angle BDX$	2) Given
3) $\angle AXC \cong \angle BXD$	3) Vertically Opposite Angles
4) $\triangle ACX \cong \triangle BDX$	4) AAS Theorem

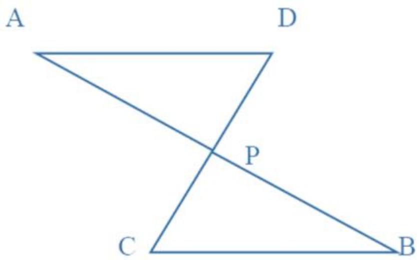
When two triangles are congruent, each part of one triangle is congruent to the corresponding part of the other triangle. That's referred to as ***corresponding parts of congruent triangles are congruent***, **thus CPCTC.**

Example 3:

Given: $\overline{AB}$ and $\overline{CD}$ bisect each other

Prove: $\overline{AD} \cong \overline{BC}$

Proof:



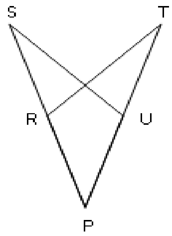
Statements	Reasons
In $\triangle APD$ and $\triangle BPC$, we have:	
1) $\overline{AB}$ bisects $\overline{CD}$	1) Given
2) P is the midpoint of $\overline{CD}$	2) Def of Bisector
3) $\overline{CP} \cong \overline{DP}$	3) Def of a midpoint
4) $\overline{CD}$ bisects $\overline{AB}$	4) Given
5) P is the midpoint of $\overline{AB}$	5) Def of Bisector
6) $\overline{AP} \cong \overline{BP}$	6) Def of a midpoint
7) $\angle APD \cong \angle BPC$	7) Vertically Opposite Angles
8) $\triangle APD \cong \triangle BPC$	8) ASA Postulate
9) $\overline{AD} \cong \overline{BC}$	9) CPCTC

Example 4:

Given: $\angle S \cong \angle T$, PR = PU, $\overline{SX} \cong \overline{TX}$

Prove: $\overline{SP} \cong \overline{TP}$

Proof:



Statements	Reasons
In □ APD and □ BPC, we have:	
1) $\angle S \cong \angle T$, $\overline{SX} \cong \overline{TX}$	1) Given
2) $\angle SXR \cong \angle TXU$	2) Vertical angles are congruent
3) $\Delta SXR \cong \Delta TXU$	3) ASA Postulate
4) $\overline{RS} \cong \overline{UT}$	4) CPCTC
5) RS=UT	5) $\cong$ segments have = lengths.
6) PR=PU	6) Given
7) PR+RS=PU+UT	7) Addition property of equality
8) PR+RS=SP PU+UT=TP	8) Segment addition property
9) SP=TP	9) Substitution
10) $\overline{SP} \cong \overline{TP}$	10) Segments with = length are $\cong$