

Complex Rational Functions

Complex fraction is a rational expression that has a fraction in its numerator, denominator or both. In other words, there is at least one small fraction within the overall fraction.

Some examples of complex fractions are: $\frac{\frac{x-9}{3x-7}}{x-1}$ and $7 + \frac{1}{x-y}$

There are two ways that you can simplify complex fractions.

Method I: Simplifying a Complex Fraction

Step 1: Find the domain of definition ($B \neq 0$; $D \neq 0$; $C \neq 0$)

Step 2: If needed, rewrite the numerator and denominator so that they are each a single fraction. In other words, you will be combining all the parts of the numerator to form one fraction and all of the parts of the denominator to form another fraction.

Step 3: Divide the numerator by the denominator.

A fraction bar is the same as a division bar where you are taking the numerator divided by the denominator. Recall that when you divide by a fraction it is the same as multiplying by its reciprocal. So in this step you will be multiplying the numerator by the reciprocal of the denominator.

$$\frac{\frac{A}{B}}{\frac{C}{D}} = \frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{A \cdot D}{B \cdot C}$$

Step 4: If needed, simplify the rational expression.

Example 1: Simplify:

$$\frac{\frac{1}{a} + \frac{3}{b}}{\frac{5}{b} - \frac{3}{a^2}} \cdot$$

$$\frac{\frac{1}{a} + \frac{3}{b}}{\frac{5}{b} - \frac{3}{a^2}} = \frac{\frac{b+3a}{ab}}{\frac{5a^2-3b}{a^2b}} = \frac{b+3a}{ab} \div \frac{5a^2-3b}{a^2b} = \frac{b+3a}{ab} \times \frac{a^2b}{5a^2-3b} = \frac{\cancel{a^2}b(b+3a)}{\cancel{ab}(5a^2-3b)} = \frac{ab+3a^2}{5a^2-3b}$$

Method II: Simplifying a Complex Fraction

Step 1: Find the domain of definition

Step 2: Multiply the numerator and denominator of the overall complex fractions by the LCD of the smaller fractions.

Step 3: If needed, simplify the rational expression.

$$\frac{\frac{A}{B}}{\frac{C}{D}} = \frac{\frac{A}{\cancel{B}} \cdot \cancel{B}D}{\frac{C}{\cancel{D}} \cdot \cancel{D}B} = \frac{A \cdot D}{C \cdot B}$$

Example 2: Simplify

$$1) \frac{\frac{3}{x-5} - \frac{4}{x+5}}{\frac{7}{x^2-25} - \frac{4}{x-5}}$$

$$\begin{aligned} & \frac{\frac{3}{x-5} - \frac{4}{x+5}}{\frac{7}{x^2-25} - \frac{4}{x-5}} && x \neq \pm 5 && \text{The LCD is } (x^2 - 25) \\ & = \frac{\left(\frac{3}{x-5} - \frac{4}{x+5}\right)(x^2 - 25)}{\left(\frac{7}{x^2-25} - \frac{4}{x-5}\right)(x^2 - 25)} && \text{Multiply up and down by } (x^2 - 25) \\ & = \frac{3(x+5) - 4(x-5)}{7 - 4(x+5)} && \text{Simplify} \\ & = \frac{3x+15-4x+20}{7-4x-20} && \text{Distribute} \\ & = \frac{-x+35}{-4x-13} && x \neq -\frac{13}{4} \end{aligned}$$

$$2) \frac{\frac{6}{2} - \frac{4x-1}{3x-8}}{\frac{x-2}{3x-8}}$$

$$\begin{aligned} & \frac{\frac{6}{2} - \frac{4x-1}{3x-8}}{\frac{x-2}{3x-8}} && x \neq 2, x \neq \frac{8}{3} && \text{The LCD is } (x-2)(3x-8) \\ & = \frac{6(x-2)(3x-8)}{\left(\frac{2}{x-2} - \frac{4x-1}{3x-8}\right)(x-2)(3x-8)} && \text{Multiply up and down by } (x-2)(3x-8) \\ & = \frac{6(x-2)(3x-8)}{2(3x-8) - (4x-1)(x-2)} && \text{Simplify} \\ & = \frac{6(x^2 - 14x + 16)}{6x - 16 - 4x^2 + 8x + x - 2} && \text{Distribute} \\ & = \frac{6x^2 - 84x + 96}{-4x^2 + 15x - 18} \end{aligned}$$

$$3) \frac{2x^{-1} + 3y^{-1}}{2x^{-1} - 3y^{-1}}$$

$$\frac{2x^{-1} + 3y^{-1}}{2x^{-1} - 3y^{-1}}$$

$$= \frac{\frac{2}{x} + \frac{3}{y}}{\frac{2}{x} - \frac{3}{y}}$$

$$x \neq 0, y \neq 0$$

Simplify the top part and the bottom part

$$= \frac{\frac{2y + 3x}{xy}}{\frac{2y - 3x}{xy}}$$

Change the dash to division

$$= \frac{2y + 3x}{xy} \div \frac{2y - 3x}{xy}$$

Change division to multiplication

$$= \frac{2y + 3x}{\cancel{xy}} \times \frac{\cancel{xy}}{2y - 3x}$$

Simplify

$$= \frac{2y + 3x}{2y - 3x} \quad x \neq \frac{2y}{3}$$