Circles in the Coordinate Plane

The equation of a circle in a coordinate plane is written knowing the coordinates of its center and the length of the radius.

Let (x, y) be a point on the circle and let (h,k) be the coordinates of the center. Using the distance formula:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\Rightarrow r=\sqrt{(x-h)^2+(y-x)^2}$$

Squaring both sides of the equation

$$\Rightarrow r^2=(x-h)^2+(y-x)^2$$

Definition 1: The equation of a circle of center (h,k) and radius r in standard form is:

$$r^2=(x-h)^2+(y-x)^2$$

In particular if the center is the origin i.e. O (0,0)

$$\Rightarrow r^2=(x-0)^2+(y-0)^2\Rightarrow r^2=x^2+y^2$$

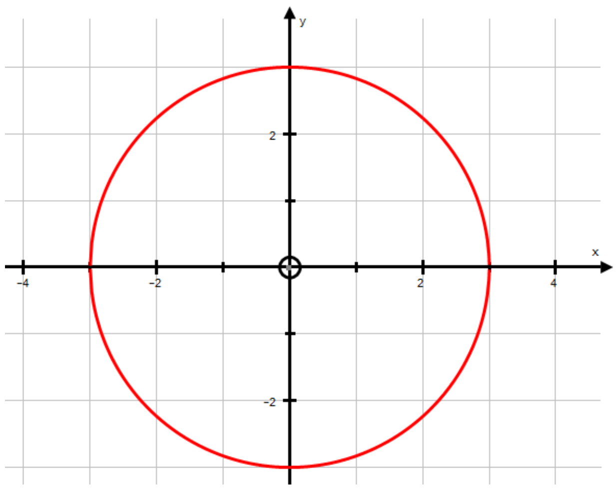
Definition 2: The unit circle is the circle whose radius is equal to 1 and whose center is the origin

i.e. $x^2+y^2=1$......... unit circle

Example 1: Find the equation of the circle whose vertex is the origin and whose radius is 3.

The vertex is the origin $\Rightarrow$(h,k) = (0,0)

$$\Rightarrow (x-h)^2+(y-x)^2=r^2$$
$$\Rightarrow (x-0)^2+(y-0)^2=3^2$$
$$\Rightarrow x^2+y^2=9$$

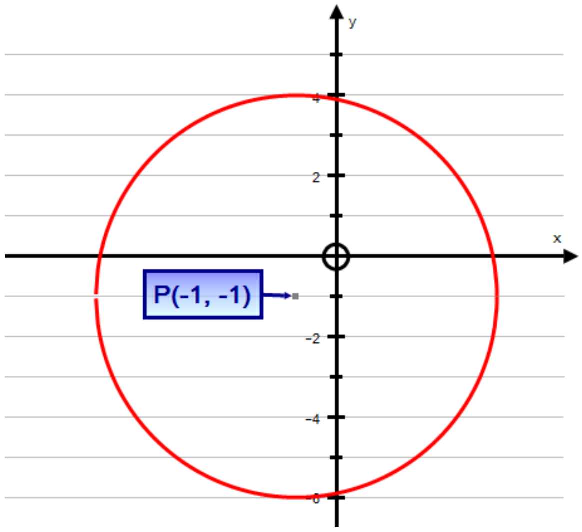


Given a point and a center, the equation of the circle can be found by substituting the values of x, y, h and x to find the value of the radius r.

Example 2: Find the equation of the circle passing through A (2, 3) and with center P (-1,-2)

The distance from the center to a point on the circle is the radius

$$
\begin{aligned}
r = d(A,P) &= \sqrt{(x_P - x_A)^2 (y_P - y_A)^2} \\
&= \sqrt{(-1-2)^2 + (-1-3)^2} \\
&= \sqrt{9+16} \\
&= \sqrt{25} \\
&= \sqrt{5}
\end{aligned}
$$



Therefore, center is P(-1,-1) and radius = r = 5

$$
\begin{aligned}
&(x-h)^2 + (y-k)^2 = r^2 \\
&(x-(-1))^2 + (y-(-1))^2 = 5^2 \\
&(x+1)^2 + (y+1)^2 = 25
\end{aligned}
$$

Graphing circles

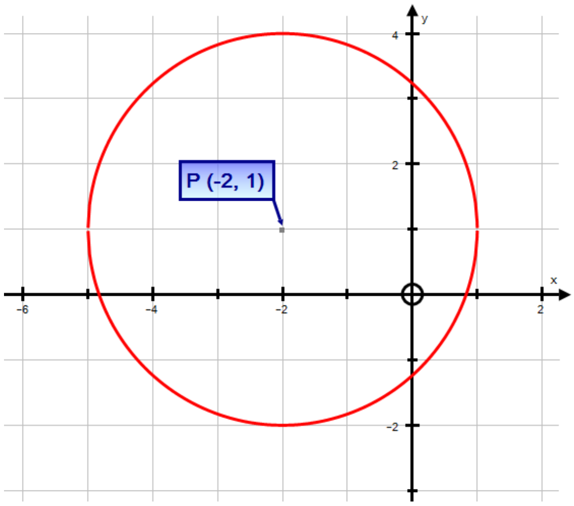
If you know the equation of a circle, you can graph the circle by identifying its center and radius.

Example 3: The equation of a circle is $(x + 2)^2 + (y - 1)^2 = 9$. Graph the circle.

Compare the given equation with $(x-h)^2 + (y-k)^2 = r^2$, h = -2 and k = 1 $\Rightarrow$ center = P(-2,1)

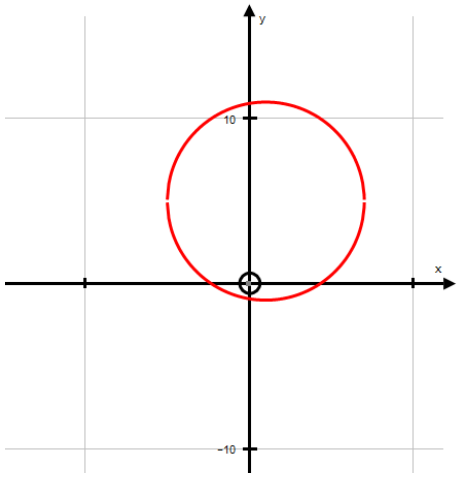
$r^2 = 9 \Rightarrow r = 3$

To graph the circle, place the point of a compass at (-2,1), set the radius at 3 units, and swing the compass to draw a full circle.



Example 4: The equation of circles is $(x-1)^2+(y-5)^2=36$

a. what are the coordinates of the center of the circles?
b. What is the length of the radius of the circle?
c. What are the coordinates of two points on the circle?



Compare the equation $(x-1)^2+(y-5)^2=36$ to the general form of the equation of a circles: $(x-h)^2+(y-k)^2=r^2$

Therefore, h = 1, k = 5, r^2 = 36, and r = 6.

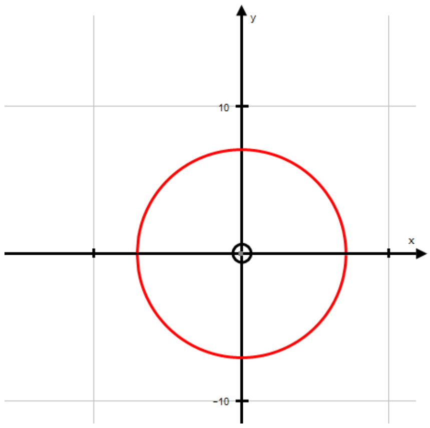
a. The coordinates of the center are (1,5).
b. The length of the radius is 6.
c. Points 6 units from (1,5) on the same horizontal line are (7,5) and (-7,5).
d. Point 6 units from (1,5) on the same vertical line are (1,11) and (1, -1).

Example 5: The equation of a circle is $x^2 + y^2 = 50$. What is the length of the radius of the circle

Compare the given equation to $x^2 + y^2 = r^2$

$r^2 = 50$
$r = \pm\sqrt{50}$
$r = \pm\sqrt{25}\sqrt{2}$
$r = \pm 5\sqrt{2}$

Since a length is always positive, r = $5\sqrt{2}$



Example 6: Find the coordinates of the points at which the line y = 2x – 1 intersects a circle with center at (0,-1) and radius of length $\sqrt{20}$.

In the equation $(x - h)^2 + (y - k)^2 = r^2$ let h = 0, k = -1, and r = $\sqrt{20}$. The equation of the circle is:

$(x-0)^2+(y-(-1))^2=(\sqrt{20})^2$ or $x^2+(y+1)^2=20$

Find the common solution of $x^2+(y+1)^2=20$ and y = 2x -1

The linear equation is solved for y $\quad x^2+(y+1)^2=20$

In terms of x. substitute, in the $\quad x^2=(2x-1+1)^2=20$

Equation of the circle, the expression $\quad x^2+(2x)^2=20$

For y and simplify the result.

Square the monomial: $\quad x^2+4x^2=20$

Write the equation in standard form: $\quad 5x^2-20=0$

Divide by the common factor, 5: $\quad x^2-4=0$

Factor the left side of the equation: $\quad (x-2)(x+2)=0$

Set each factor equal to zero: $\quad x-2=0 \qquad x+2=0$

Solve each equation for x: $\quad x=2 \qquad x=-2$

For each value of x find the $\quad y=2x-1 \qquad y=2x-1$

Corresponding value of y: $\quad y=2(2)-1 \qquad y=2(-2)-1$

$y=3 \qquad y=-5$