Arcs and Chords

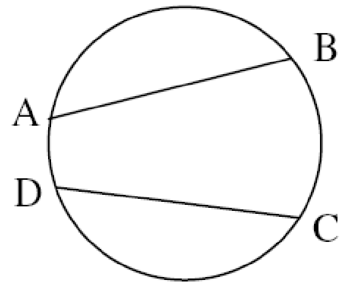
Arc – Chord Theorem: In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

- In a circle, 2 minor arcs are $\cong \Rightarrow$ corresponding chords are $\cong$.
- In a circle, 2 chords are $\cong \Rightarrow$ corresponding minor arcs are $\cong$.

If $\overline{AB} \cong \overline{CD} \Rightarrow AB \cong CD$

and

if $AB \cong CD \Rightarrow \overline{AB} \cong \overline{CD}$



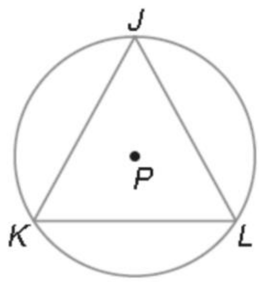
Example 1: The vertices of equilateral triangle *JKL* are located on a circle with center P. Identify all congruent minor arcs.

Given that □ *JKL* is equilateral

$\Rightarrow \overline{JK} \cong \overline{KL} \cong \overline{JL}$ (Def. of an equilateral triangle)

$\Rightarrow JK \cong KL \cong JL$ (Minor arcs are $\cong \Rightarrow$ corresponding chords are $\cong$)

Therefore, the congruent minor arcs are: $JK, KL, and\ JL$

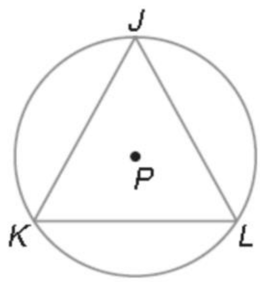


Example 2: Proof of Theorem 1

Given: Circle with center X and radius $\overline{XV}$

$UV \cong YW$

Prove: $\overline{UV} \cong \overline{YW}$



Proof:

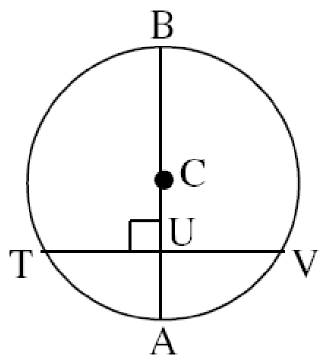
Statements	Reasons
1) $UV \cong YW$	1) Given
2) $\angle UXV \cong \angle WXY$	2) If 2 arcs are $\cong$ $\Rightarrow$ central angles are $\cong$
3) $\overline{UX} \cong \overline{VX} \cong \overline{WX} \cong \overline{YX}$	3) Radii of the same circles are $\cong$
4) $\triangle UXV \cong \triangle WXY$	4) SAS theorem
5) $\overline{UV} \cong \overline{YW}$	5) CPCTC

Radius – Chord Theorem: In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.

$\overline{BA}$ is a diameter and $\overline{TV}$ is a chord

$\overline{BA} \perp \overline{TV}$ at U $\Rightarrow \overline{TU} \cong \overline{UV}$ and

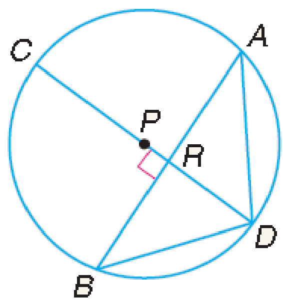
$TA \cong AV$



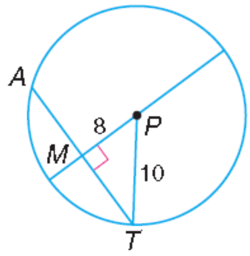
Converse of Radius – Chord Theorem: In a circle, a diameter bisects a chord and its arc if and only if it is perpendicular to the chord.

$\overline{AR} \cong \overline{BR}$ and $\overline{AD} \cong \overline{BD}$ if and only if

$\overline{CD} \perp \overline{AB}$



Example 3: In the circle with center P, if $\overline{PM} \perp \overline{AT}$, PT = 10, and PM = 8, find AT.



□ PMT is a right angle. Def. of perpendicular

□ PMT is a right triangle. Def. of right triangle

$(MT)^2 + (PM)^2 = (PT)^2$ Pythagorean Theorem

$(MT)^2 + 8^2 = 10^2$ Replace PM with 8 and PT with 10.

$(MT)^2 + 64 = 100$ $8^2 = 64$; $10^2 = 100$

$(MT)^2 + 64 - 64 = 100 - 64$ Subtract 64 from each side.

$(MT)^2 = 36$ Simplification

$\sqrt{(MT)^2} = \sqrt{36}$ Take the square root of each side.

MT = 6 Simplification

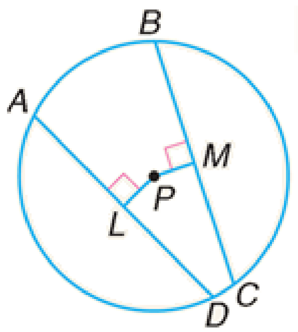
$\overline{PM}$ bisects $\overline{AT}$. Therefore, AT=2(MT). So, AT=2(6)=12.

Chord – Distance Theorem: In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

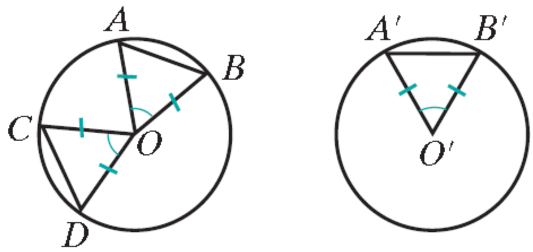
$$\overline{AD} \cong \overline{BC} \Leftrightarrow \overline{LP} \cong \overline{PM}$$

$$\overline{LP} \perp \overline{AD}$$

$$\overline{PM} \perp \overline{BC}$$



Example 4:



Given: $C(O,OA) \cong C'(O',O'A')$ and $\angle COD \cong \angle AOB \cong \angle A'O'B'$

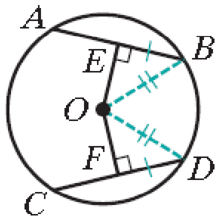
Prove: $\overline{CD} \cong \overline{AB} \cong \overline{A'B'}$

Proof:

Statements	Reasons
In $\triangle AOB$ and $\triangle COD$, we have:	
1) $\angle COD \cong \angle AOB$	1) Given
2) $\overline{OA} \cong \overline{OB} \cong \overline{OC} \cong \overline{OD}$	2) Radii of the same circle are congruent
3) $\triangle AOB \cong \triangle COD$	3) SAS theorem
4) $\overline{CD} \cong \overline{AB}$	4) CPCTC
In $\triangle A'O'B'$ and $\triangle COD$, we have:	
5) $\angle COD \cong \angle A'O'B'$	5) Given
6) $\overline{O'A'} \cong \overline{O'B'} \cong \overline{OC} \cong \overline{OD}$	6) Radii of the congruent circles are congruent
7) $\triangle A'O'B' \cong \triangle COD$	7) SAS theorem
8) $\overline{CD} \cong \overline{A'B'}$	8) CPCTC
9) $\overline{CD} \cong \overline{A'B'} \cong \overline{AB}$	9) Transitive property

Corollary: In a circle or in congruent circles, two chords are congruent if and only if their central angles are congruent.

Example 5:



Given: $C(O,OA)$ *with* $\overline{AB} \cong \overline{CD}$, $\overline{OE} \perp \overline{AB}$, *and* $\overline{OF} \perp \overline{CD}$,

Prove: $\overline{OE} \cong \overline{OF}$

Proof:

Statements	Reasons
1) $\overline{OE} \perp \overline{AB}$	1) Given
2) $\overline{OE}$ bisects $\overline{AB}$	2) A line passing through the center and $\perp$ to the chord bisects the chord and its arcs.
3) $\overline{EB} \cong \overline{EA}$	3) Def of a bisector
4) $\overline{OF} \perp \overline{CD}$	4) Given
5) $\overline{OF}$ bisects $\overline{CD}$	5) A line passing through the center and $\perp$ to the chord bisects the chord and its arcs.
6) $\overline{FD} \cong \overline{FC}$	6) Def of a bisector
7) But $\overline{AB} \cong \overline{CD}$	7) Given
8) $\overline{FD} \cong \overline{EB}$	8) Substitution
9) $\overline{OB} \cong \overline{OD}$	9) Radii of the same circle
10) $\triangle OBE \cong \triangle ODF$	10) HL Theorem
11) $\overline{OE} \cong \overline{OF}$	11) CPCTC

Corollary: In a circle, if the lengths of two chords are unequal, then the shorter chord is farther from the center.