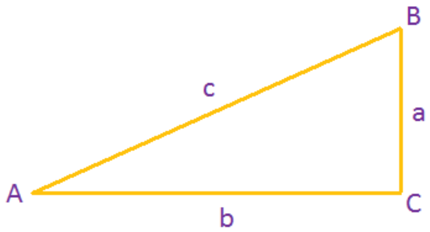
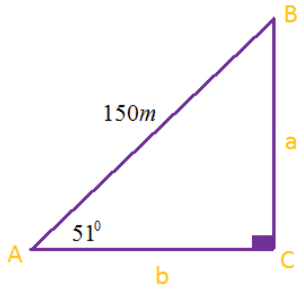
Applications and Models

Part A: Applications involving right triangles

The three angles of a right triangle are denoted by the letters A, B, and C(where C is the right angle), and the lengths of the sides opposite to these angles by the letters a, b, and c (where c is the hypotenuse).



Example 1: Solve the right triangle shown below rounding all measurements to one decimal place (if necessary).



Identify the given information: $\angle A = 51^0; \angle C = 90^0; c = 150m$

Identify what needs to be found: $\angle B;\ a;\ and\ b$

1) Find $\angle B$:

$$m\angle A + m\angle B + m\angle C = 180^0$$
$$\Rightarrow 51^0 + m\angle B + 90^0 = 180^0$$
$$\Rightarrow m\angle B = 39^0$$

2) Find a

We know angle A and the hypotenuse, so the sine function can be used to determine the length of side a.

$$\sin A = \frac{a}{c}$$
$$\Rightarrow \sin 51^0 = \frac{a}{150}$$
$$\Rightarrow a = 150\left(\sin 51^0\right)$$
$$\therefore a \approx 116.6m$$

3) **Find b**

Side b is adjacent to angle A and since we know the measurements of angle A and the hypotenuse the cosine function can now be used to find b.

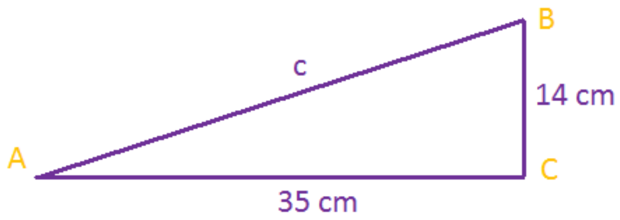
$$\cos A = \frac{b}{c}$$

$$\Rightarrow \cos 51^0 = \frac{b}{150}$$

$$\Rightarrow b = 150\left(\cos 51^0\right)$$

$$\therefore a \approx 94.4m$$

Example 2: Solve the right triangle shown below rounding all measurements to one decimal place (if necessary).



Identify the given information: $\angle C = 90^0;\ a = 14\ cm;\ and\ b = 35\ cm$

Identify what needs to be found: $\angle A;\ \angle B;\ and\ c$

1) Find $\angle A$:

Since we know the measurements of sides a and b we can use the tangent function to find the measurement of angle A.

$$\tan A = \frac{a}{b}$$

$$\Rightarrow \tan A = \frac{14}{35}$$

$$\Rightarrow A \approx 21.8^0$$

2) Find $\angle B$

$$m\angle A + m\angle B + m\angle C = 180^0$$

$$\Rightarrow 21.8^0 + m\angle B + 90^0 = 180^0$$

$$\Rightarrow m\angle B = 68.2^0$$

3) Find c

Since we know the lengths of the sides a and b the Pythagorean Theorem can be used to find the length of the hypotenuse. You do not want to use a trigonometric function involving either of the two angles we just found because they have been rounded off. Using the original information given and the Pythagorean Theorem will provide a much better estimate of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$

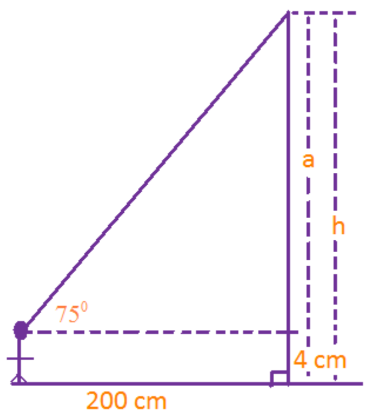
$$\Rightarrow (14)^2 + (35)^2 = c^2$$

$$\Rightarrow c^2 = 196 + 1225 = 1421$$

$$\Rightarrow c^2 = 1421$$

$$\Rightarrow c \approx 37.7 \ \ cm$$

Example 3: A child who is 4 feet tall is standing 200 cm away from the center of the base of the Emirates Towers. If the child must look up at an angle of 75° to see the top of the tower, how tall is the Emirates Towers?



1) Find a

For the triangle involving a, we know both the measurements of angle A and the length of the side adjacent to it. Therefore, we can use the tangent function to determine a.

$$\tan A = \frac{a}{b}$$

$$\Rightarrow \tan 75^0 = \frac{a}{200}$$

$$\Rightarrow a = 200\left(\tan 75^0\right)$$

$$\Rightarrow a \approx 746 \ cm$$

2) Find h

The height of the Emirates Tower is equal to the value of a plus the height of the child.

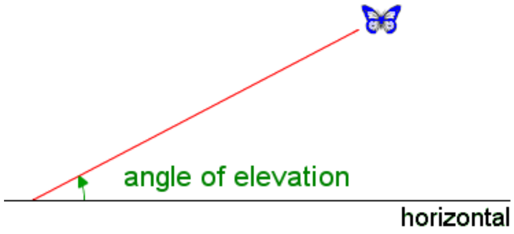
$$\text{h=4+a}$$

$$\Rightarrow \text{h} \approx 4+746$$

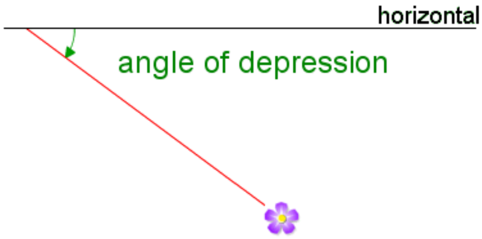
$$\Rightarrow \text{h} \approx 750$$

Angles of elevation and depression

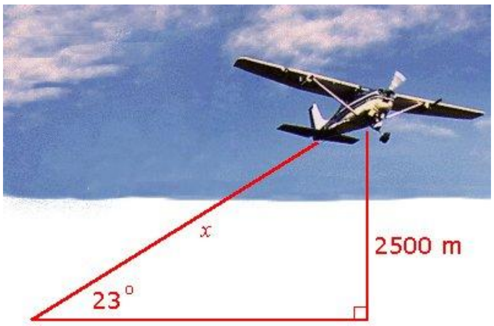
In surveying, the **angle of elevation** is the angle from the horizontal looking **up** to some object:



The **angle of depression** is the angle from the horizontal looking **down** to some object:



Example 4: The angle of elevation of an airplane is 23°. If the airplane's altitude is 2500 m, how far away is it?



Let the distance be *x*.

$$\sin 23^0 = \frac{2500}{x}$$

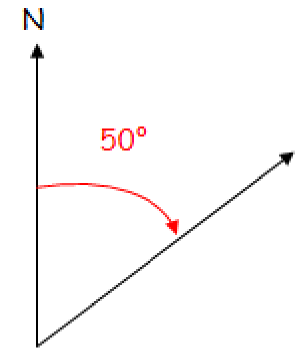
$$\Rightarrow x = \frac{2500}{\sin 23^0} = 6400$$

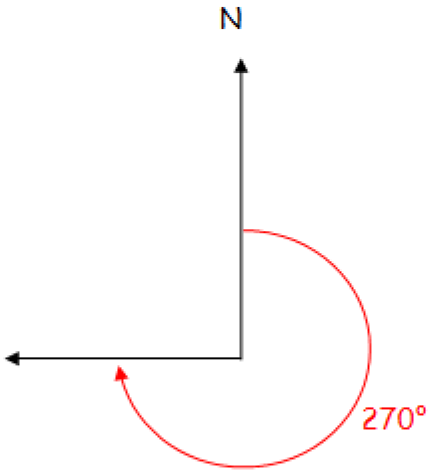
Application involving Bearings

In surveying and navigation, directions are generally given in terms of bearings. A bearing measures the acute angle that a path or line sight makes with a fixed north – south line.

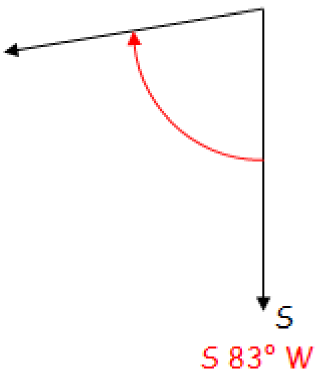
There are two methods for expressing bearing.

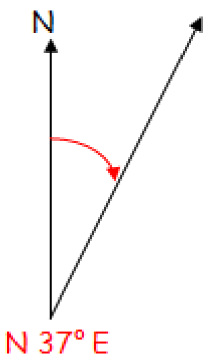
1) Single Angle Given: When a single angle is given, it is understood that the bearing is measured in a **clockwise** direction from **due north**. In general in air navigation, bearings are measured in degrees clockwise from north.



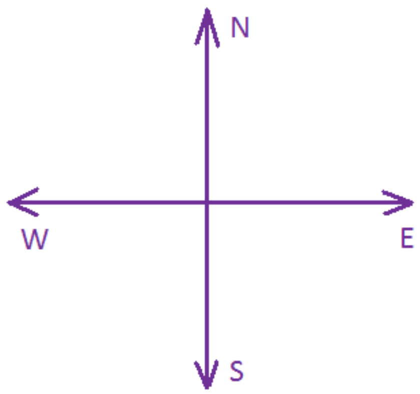


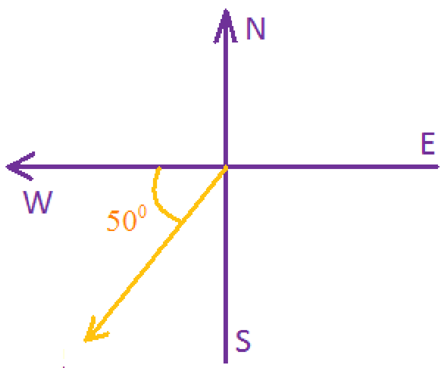
2) The second method starts with a north-south line and uses an acute angle to show the direction, either east or west, from this line.



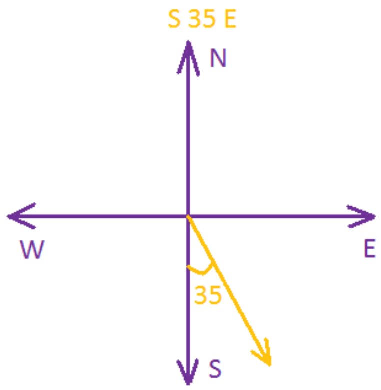


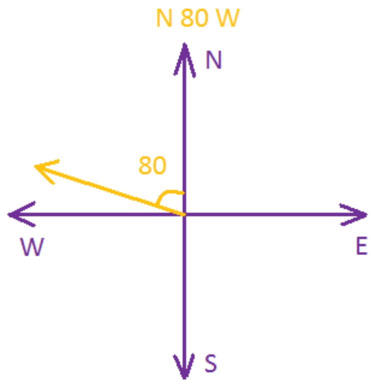
Example 5: W 50^0 S means that you face west, and then veer off 50^0 to the south.



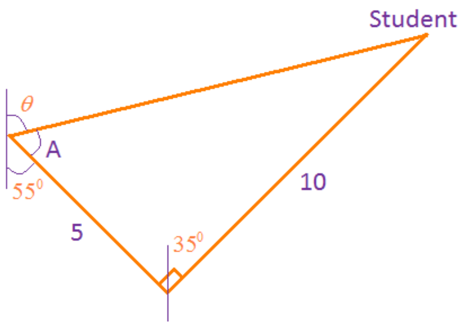


Other examples:





Example 6: A student leaves the campus on a bearing of S 55° E and travels 5 miles before turning on a bearing of N 35° E. After traveling 10 miles on this new bearing what is the bearing of the student from the campus?



Draw diagram of the situation
Identify known information: Right triangle, Adjacent side (b) = 5, Opposite side (a) = 10
Identify what needs to be found: The angle A; The angle θ

1) Find angle A

Since the adjacent and opposite sides are known the tangent function is used to find the measurement of angle A

$$\tan A = \frac{a}{b}$$

$$\Rightarrow \tan A = \frac{10}{5}$$

$$\Rightarrow A \approx 63.4^0$$

2) Find angle θ

Angle θ, A and the 55° angle form a straight line and are therefore supplementary angles whose sum must equal 180°.

$$m\angle\theta + m\angle A + 55^0 = 180^0$$

$$\Rightarrow m\angle\theta + 63.4^0 + 55^0 = 180^0$$

$$\Rightarrow m\angle\theta = 61.6^0$$

The student's bearing from the campus is N 61.6^0 E.