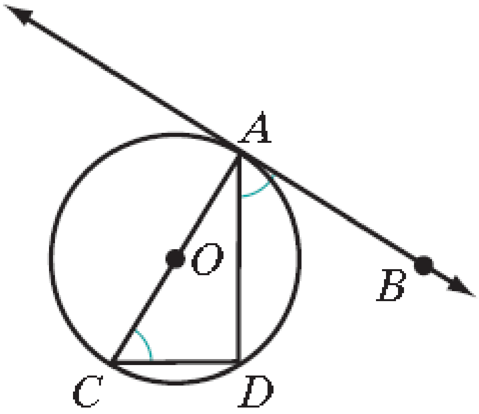
Angles Formed by Tangents, Chords, and Secants

In the diagram, $\overleftrightarrow{AB}$ is tangent to the circle of center O at A, $\overline{AD}$ is a chord, and $\overline{AC}$ is a diameter. When $\overline{CD}$ is drawn, $\angle ADC$ is a right angle because it is an angle inscribed in a semicircle, and $\angle ACD$ is the complement of $\angle CAD$. Also, $\overline{CA} \perp \overline{AB}$, $\angle BAC$ is a right angle, and $\angle DAB$ is the complement of $\angle CAD$. Therefore, since complements of the same angle are congruent, $\angle ACD \cong \angle DAB$.



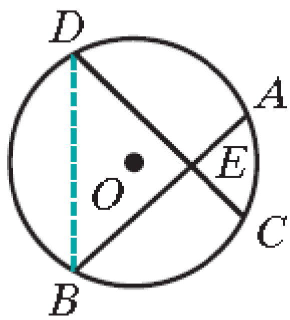
We can conclude that since $m\angle ACD = \frac{1}{2} mAD$, then

$$m\angle DAB = \frac{1}{2} mAD$$

Theorem 1: The measure of an angle formed by a tangent and a chord that intersect at the point of tangency is equal to one – half the measure of the intercepted arc.

Angles Formed by Two Intersecting Chords

Two chords $\overline{AB}$ and $\overline{CD}$ intersect in the interior of the circle with center O and $\overline{DB}$ is drawn. $\angle AED$ is an exterior angle of $\triangle DEB$. Therefore,



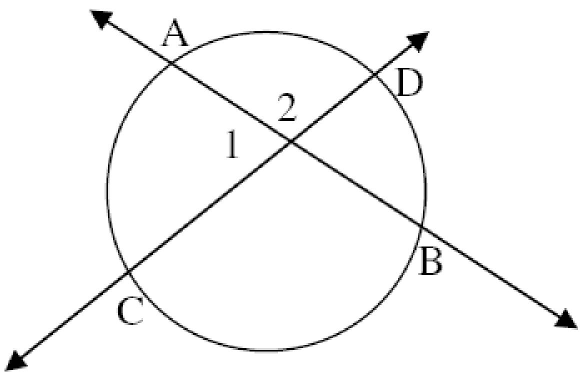
$$\begin{aligned} m\angle AED &= m\angle BDE + m\angle DBE \\ &= \frac{1}{2} mBC + \frac{1}{2} mDA \\ &= \frac{1}{2}\left(mBC + mDA\right) \end{aligned}$$

BC is the arc intercepted by $\angle BEC$ and DA is intercepted by $\angle AED$, the angle vertical to $\angle BEC$.

Theorem 2: The measure of an angle formed by two chords intersecting within a circle is equal to one – half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

$$m\angle 1 = \left(\frac{mAC + mDB}{2}\right)$$

$$m\angle 2 = \left(\frac{mAD + mBC}{2}\right)$$



Angles Formed by Tangents and Secants

We will study in this section three cases:

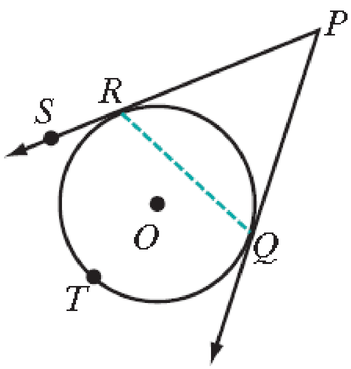
1- Angles formed by two tangents
2- Angles formed by a tangent and a secant
3- Angles formed by two secants

All of the angles formed in each of the three cases have vertices outside the circle, are related to the measures of the intercepted arcs.

1- Angles formed by two tangents

$\overleftrightarrow{PRS}$ is tangent to the circle of center O at R, $\overleftrightarrow{PQ}$ is tangent to the circle at Q, and T is a point on the major arc RQ. Chord $\overline{RQ}$ is drawn. Then $\angle SRQ$ is an exterior angle of $\triangle RQP$.

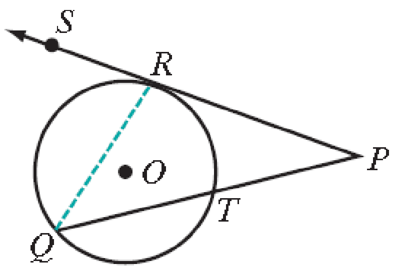
$$m\angle PQR + m\angle P = m\angle SRQ$$
$$\Rightarrow m\angle P = m\angle SRQ - m\angle PQR$$
$$\Rightarrow m\angle P = \frac{1}{2}mRTQ - \frac{1}{2}mRQ$$
$$\Rightarrow m\angle P = \frac{1}{2}\left(mRTQ - mRQ\right)$$



2- Angles formed by a tangent and a secant

$\overleftrightarrow{PRS}$ is tangent to the circle of center O at R, $\overleftrightarrow{PTQ}$ is a secant that intersects the circle at Q and T. Chord $\overline{RQ}$ is drawn. Then $\angle SRQ$ is an exterior angle of $\triangle PRQ$.

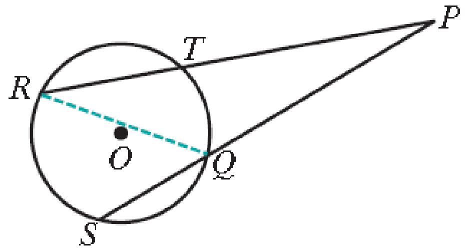
$$m\angle RQP + m\angle P = m\angle SRQ$$
$$\Rightarrow m\angle P = m\angle SRQ - m\angle RQP$$
$$\Rightarrow m\angle P = \frac{1}{2}mRQ - \frac{1}{2}mRT$$
$$\Rightarrow m\angle P = \frac{1}{2}\left(mRQ - mRT\right)$$



3- Angles formed by two secants

$\overleftrightarrow{PTR}$ is a secant to the circle of center O that intersects the circle at R and T, $\overleftrightarrow{PQS}$ is a secant that intersects the circle at Q and S. Chord $\overline{RQ}$ is drawn. Then $\angle RQS$ is an exterior angle of $\triangle RQP$.

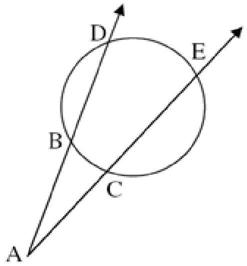
$$m\angle PRQ + m\angle P = m\angle RQS$$
$$\Rightarrow m\angle P = m\angle RQS - m\angle PRQ$$
$$\Rightarrow m\angle P = \frac{1}{2} mRS - \frac{1}{2} mQT$$
$$\Rightarrow m\angle P = \frac{1}{2}\left(mRS - mQT\right)$$



For each pair of lines, a tangent and a secant, two secants, an two tangents, the steps necessary to prove the following theorem have been given:

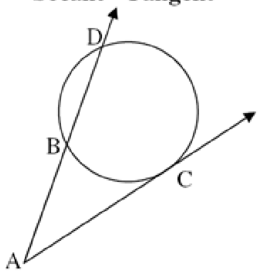
Theorem 3: The measure of an angle formed by a tangent and a secant, two secants, two secants, or two tangents intersecting outside the circle is equal to one half the difference of the measures of the intercepted arcs.

Two secants



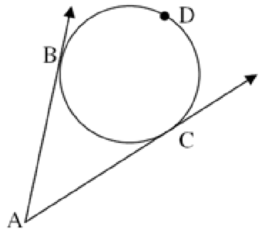
$$m\angle DAC = \frac{1}{2}\left(mDE - mBC\right)$$

Secant - Tangent



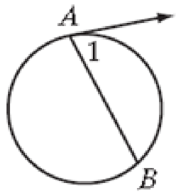
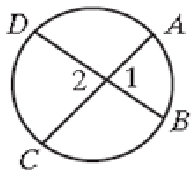
$$m\angle DAC = \frac{1}{2}\left(mDC - mBC\right)$$

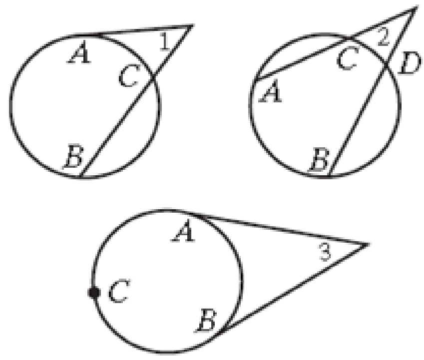
Two Tangents



$$m\angle BAC = \frac{1}{2}\left(mBDC - mBC\right)$$

Check the summary table below:

Type of Angle	Degree Measure	Example
Formed by a Tangent and a Chord	The measure of an angle by a tangent and a chord that intersect at the point of tangency is equal to one – half the measure of the intercepted arc.	 $m\angle 1 = \frac{1}{2} mAB$
Formed by Two Intersecting Chords	The measure of an angle formed by two intersecting chords is equal to one – half the sum of the measure of the arcs intercepted by the angle and its vertical angle.	 $m\angle 1 = \frac{1}{2}\left(mAB + mCD\right)$ $m\angle 2 = \frac{1}{2}\left(mAB + mCD\right)$

Formed by Tangents and Secants	The measure of an angle formed by a tangent and a secant, two secants, or two tangents intersecting outside the circle is equal to one – half the difference of the measures of the intercepted arcs.	 $m\angle 1 = \frac{1}{2}\left(mAB - mAC\right)$ $m\angle 2 = \frac{1}{2}\left(mAB - mCD\right)$ $m\angle 3 = \frac{1}{2}\left(mACB - mAB\right)$