Absolute Value Inequalities

Now let's look at how absolute values work in inequalities. Consider the inequality $|x| < 3$.
Numbers like 1, 2, 1.5, and 2.9 would work, but numbers like –1, -2, -1.5, and –2.9 would also work, since their magnitude is also less than three. We can also think of this problem as asking what numbers are within 3 units of zero on the number line. This is going to extend both left and right from zero.

From this, we can conclude that $|x| < 3$ can be rewritten as $\{x \mid -3 < x < 3\}$, since *x* is less than three, but if it were smaller than –3, the *magnitude* would no longer be less than three.

Example 1: Rewrite $|x| \geq 4$ as an inequality without using absolute values.

In this case, the magnitude of *x* need to be greater than or equal to 4, so *x* needs to be 4 units or further away from zero on a number line. So *x* could be greater than 4, or *x* could be less than –4, since the magnitude would then be larger than 4.
So $|x| \geq 4$ can be rewritten as $\{x \mid x \geq 4 \quad or \quad x \leq -4\}$

Rule 1: *A* and *B* can be variables, numbers, or expressions

1) $|A| < B$ can be rewritten as $-B < A < B \;\; (-B < A \quad and \quad A < B)$
2) $|A| > B$ can be rewritten as $A > B \quad or \quad A < -B$

Example 2: Rewrite $|x-2| < 5$ without absolute values.

Using the first rule, we can rewrite this as $-5 < x-2 < 5$
Notice that we're saying that the quantity we're taking the absolute value of is less than 5, so the quantity is less than 5 and greater than –5.

Remark: If a problem involves absolute values, but the absolute value is not by itself on one side of the inequality, we first want to use our basic inequality rules to isolate the absolute value. In other words, we want to get the absolute value by itself on one side of the inequality before rewriting it.

Example 3: Solve $-|x-2| + 8 < 3$

$-\|x-2\| + 8 < 3$	Subtract 8 from both sides
$-\|x-2\| < -5$	Divide by -1. Don't forget to change the direction of the inequality.
$\|x-2\| > 5$	Now we can rewrite the inequality with the absolute value
$x-2 > 5 \quad or \quad x-2 < -5$	Add 2 to both sides of each inequality
$x > 7 \quad or \quad x < -3$	

In set notation, the solution is: $\{x \mid x > 7 \quad or \quad x < -3\}$

Example 4: Solve: $2|x-1|>6$

$2|x-1|>6$

$\Rightarrow |x-1|>3$

$\Rightarrow x-1>3 \quad or \quad -(x-1)>3$

$\Rightarrow x-1>3 \quad or \quad -x+1>3$

$\Rightarrow x-1+1>3+1 \quad or \quad -x+1-1>3-1$

$\Rightarrow x>4 \quad or \quad -x>2$

$\Rightarrow x>4 \quad or \quad x<-2$

The solution is: $\{x \mid x>4 \quad or \quad x<-\}$

Example 5: Solve and graph: |2x - 5| < 13

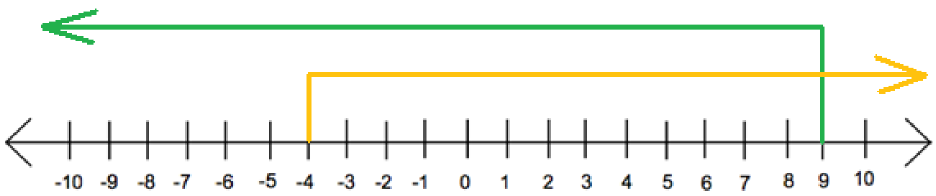
2x - 5 < 13 and -(2x - 5) < 13

2x < 18 -2x + 5 < 13

x < 9 -2x < 8

x > -4

The algebraic solution is x < 9 and x > -4.



Example 6: Solve and graph: |9 + 3x| + 12 ≥ 48

|9 + 3x| ≥ 36 *(subtracted 12 on both sides)*

9 + 3x ≥ 36 or -(9 + 3x) ≥ 36

3x ≥ 27 -9 - 3x ≥ 36

x ≥ 9 -3x ≥ 36

x ≤ -12

The algebraic solution is x ≥ 9 or x ≤ -12.