Absolute Value Equations

Definition 1: If x is a real number, then $|x|$ is called the absolute value of x and it is the distance between x and 0.

For example, $|0|=0, |3|=3 \& |-5|=5$

Now consider the distance between -2 and 3 is either $|3-(-2)|=|5|=5$. Thus if a and b are any two numbers, then $|a-b|=|b-a|$

Many equations and inequalities involving absolute values can be solved using this geometric aspect of absolute value .

Solving equations : To solve an equation involving absolute value

i. Isolate the absolute value one side of the equation.
ii. Set its contents equal to both + and - the other side of the equation.
iii. Now solve both equations.

Example 1: Solve
$2|x|+5=8$

Solution: i) Isolating absolute value as

$2|x|+5=8$

$2|x|+5-5=8-5$

$2|x|=3$

$|x|=\frac{3}{2}$

ii) Set the contents of the absolute value equal to $+\frac{3}{2}$ and $-\frac{3}{2}$

i. e. x = $+\frac{3}{2}$ and x = $-\frac{3}{2}$

Example 2: Solve: $6|2x+3|-5=13$

i) Isolate the absolute value as

$6|2x+3|-5=13$

$6|2x+3|=18$

$|2x+3|=3$

ii) Setting the contents of the absolute value equal to +3 and -3 we get,

$2x+3=3 \quad and \quad 2x+3=-3$

$2x=0 \quad and \quad 2x=-6$

$x=0 \quad and \quad x=-3$

x = 0 and x = - 3

Example 3: Solve: $7|4x-1|=0$

$7|4x-1|=0$

$\Rightarrow |4x-1|=0$

$\Rightarrow 4x-1=0$

$\Rightarrow 4x=1$

$\Rightarrow x=\frac{1}{4}$

Example 4: Solve: $2|x+3|+4=2$ if possible

$2|x+3|+4=2$

$\Rightarrow 2|x+3|=2-4$

$\Rightarrow 2|x+3|=-2$

$\Rightarrow |x+3|=-1$

Look carefully at the equation $|x+3|=-1$. The absolute value of any number is greater or equal to zero, so how can we obtain a result of -1?

Therefore, $|x+3|=-1$is impossible. There is no solution for $2|x+3|+4=2$

Example 5: Solve: $|2x-4|=|4x-16|$ if possible

$|2x-4|=|4x-16|$

$\Rightarrow 2x-4=4x-16 \quad and \quad 2x-4=-(4x-16)$

$\Rightarrow -2x=-12 \quad and \quad 6x=20$

$\Rightarrow x=6 \quad and \quad x=\frac{10}{3}$

Example 6: Solve |4 + x| - 2x = 7 - 5x .

|4 + x| = 7 - 3x add 2x to isolate

4 + x = 7 - 3x or - (4 + x) = 7 - 3x separate into 2 equations

4 + 4x = 7 -4 - x = 7 - 3x solve each equation

4x = 3 -4 + 2x = 7

x = 3/4 2x = 11

x = 11/2

CHECK:

for x = $\frac{3}{4}$ $\left|4+\frac{3}{4}\right|-2\left(\frac{3}{4}\right)=7-5\left(\frac{3}{4}\right)$

$$\left|\frac{19}{4}\right|-\frac{3}{2}=7-\frac{15}{4}$$

$$\frac{19}{4}-\frac{3}{2}=7-\frac{15}{4}$$

$$\frac{13}{14}=\frac{13}{14}$$

for x = $\frac{11}{2}$, $\left|4+\frac{11}{2}\right|-2\left(\frac{11}{2}\right)=7-5\left(\frac{11}{2}\right)$

$$\left|\frac{19}{2}\right|-11=7-\frac{55}{2}$$

$-\frac{3}{2}\neq-\frac{41}{2}$ Therefore, $\frac{11}{2}$ is not part of your solution.

The final solution is x = 3/4.

Example 7: Solve $9-|x-2|=7$

$9-|x-2|=7$

$9-7=|x-2|$

$2=|x-2|$

$|x-2|=2$

$(x-2)=2$ or $(x-2)=-2$

$x=4$ $x=0$